Aufgabe 1: Berechnen Sie die folgenden Summen und Differenzen.

a) $\frac{17}{128} - \frac{33}{128} = \frac{17-33}{128} = \frac{-16}{128} = -\frac{4}{32} = -\frac{1}{8}$

b) $\frac{n}{5m} + \frac{2n}{5m} = \frac{n+2n}{5m} = \frac{3n}{5m}$

c) $\frac{6}{3k} - \frac{a+d}{3k} = \frac{6-(a+d)}{3k} = \frac{6-a-d}{3k}$

$5 = 5 \cdot \frac{2xy}{2xy}$

d) $5 + \frac{3}{2xy} = \frac{10xy+3}{2xy}$

e) $\frac{a}{a+1} - \frac{-1}{a+1} = \frac{a-(-1)}{a+1} = \frac{a+1}{a+1} = 1$

Aufgabe 2: Berechnen Sie die folgenden Summen und Differenzen.

a) $\frac{2x}{3y^6} + \frac{x}{4y^3} + \frac{1}{2y^6} = \frac{4\cdot 2x}{12y^6} + \frac{3y^3\cdot x}{12y^6} + \frac{6\cdot 1}{12y^6} = \underline{\frac{8x+3y^3x+6}{12y^6}}$

$$\left.\begin{aligned} 3y^6 &= \underline{3\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y} \\ 4y^3 &= \underline{2\cdot 2}\cdot y\cdot y\cdot y \\ 2y^6 &= 2\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y \end{aligned}\right\} \text{kgV} = 3\cdot y^6\cdot 2^2 = 12y^6$$

b) $\frac{ab}{c-d} + \frac{1-2ab}{d-c} + 1 = \frac{ab}{c-d} + \frac{1-2ab}{(-1)\cdot(c-d)} + \frac{c-d}{c-d} = \frac{ab}{c-d} + \frac{\cancel{(-1)}\cdot(2ab-1)}{\cancel{(-1)}\cdot(c-d)} + \frac{c-d}{c-d}$

$(c-d) = (-1)\cdot(d-c)$

$$= \frac{ab+2ab-1+c-d}{c-d}$$

$$= \frac{3ab+c-d-1}{c-d}$$

c) $\frac{3p}{p^2-q^2} - \frac{4q}{p+q} - \frac{p+q}{p-q} = \frac{3p}{p^2-q^2} - \frac{4q(p-q)}{p^2-q^2} - \frac{(p+q)^2}{p^2-q^2} = \frac{3p-4pq+4q^2-(p+q)^2}{p^2-q^2}$

$(p+q)(p-q) = p^2 - \cancel{pq} + \cancel{pq} - q^2 = p^2-q^2$

$$= \frac{3p-4pq+4q^2-(p+q)(p+q)}{p^2-q^2}$$

$$= \frac{3p-4pq+4q^2-p^2-pq-pq-q^2}{p^2-q^2}$$

$$= \underline{\frac{-p^2+3q^2+3p-6pq}{p^2-q^2}}$$

d) $\frac{-3a^2d}{c(3a^2-6a+3)} - \frac{da+d}{ca-c} + \frac{3}{a-1}$

$c(3a^2-6a+3) = 3c(a^2-2a+1) = \underline{3c(a-1)(a-1)} = \text{kgV}$

$ca-c = c(a-1)$

$a-1$

$$\frac{-3a^2d}{3c(a-1)(a-1)} - \frac{3\cdot(a-1)\cdot(da+d)}{3c(a-1)(a-1)} + \frac{3c(a-1)\cdot 3}{3c(a-1)(a-1)}$$

$$= \frac{-\cancel{3}a^2d - \cancel{3}(a-1)\cdot d(a+1) + \overset{3}{\cancel{9}}c(a-1)}{\cancel{3}c(a-1)(a-1)} = \underline{\frac{-a^2d - d(a-1)(a+1) + 3c(a-1)}{c(a-1)(a-1)}}$$

Aufgabe 3: Multiplizieren Sie die folgenden Brüche und kürzen Sie soweit möglich.

a) $\frac{3x}{8y} \cdot \frac{9z}{6y} = \frac{3x \cdot 9z}{8y \cdot 6y} = \frac{27xz}{48y^2} = \frac{9xz}{16y^2}$

b) $\frac{-4xy}{2y^2} \cdot \frac{3y}{-6} = \frac{(-4xy) \cdot 3y}{2y^2 \cdot (-6)} = \frac{\cancel{-12}x\cancel{y^2}}{\cancel{-12}\cancel{y^2}} = x$

$\frac{+4x\cancel{y} \cdot 3\cancel{y}}{2\cancel{y^2} \cdot (+6)} = \frac{12x}{12} = x$

c) $(4w-4) \cdot \frac{v}{w-1} = \frac{(4w-4) \cdot v}{w-1} = \frac{4 \cdot \cancel{(w-1)} \cdot v}{\cancel{(w-1)}} = 4v$

d) $\frac{\cancel{a}}{3f+6g} \cdot \frac{9f-6g}{a^{\cancel{2}}} = \frac{9f-6g}{(3f+6g) \cdot a} = \frac{\cancel{3} \cdot (3f-2g)}{\cancel{3} \cdot (f+2g) \cdot a} = \frac{3f-2g}{(f+2g) \cdot a}$

e) $\frac{a-b}{\cancel{(a+b)}} \cdot \frac{\cancel{(a+b)}}{2a-2b} = \frac{\cancel{(a-b)}}{2\cancel{(a-b)}} = \frac{1}{2}$

Aufgabe 4: Vervollständigen Sie die beiden "Mauern". Ein Backstein entspricht der Summe der beiden unteren Backsteine.

		$\frac{6}{a}$		
	$\frac{5}{2a}$		$\frac{7}{2a}$	
$\frac{1}{a}$		$\frac{3}{2a}$		$\frac{2}{a}$

		$-\frac{3x}{9x}$		
	$\frac{-1}{3x}$		$\frac{1-x}{3x}$	
$\frac{x-1}{3x}$		$-\frac{1}{3}$		$\frac{1}{3x}$

$$\frac{2\cdot 1}{2a} + \frac{3}{2a} = \frac{2+3}{2a} = \frac{5}{2a}$$

$$\frac{3}{2a} + \frac{2\cdot 2}{2a} = \frac{3+4}{2a} = \frac{7}{2a}$$

$$\frac{5}{2a} + \frac{7}{2a} = \frac{12}{2a} = \frac{6}{a}$$

$$\frac{x-1}{3x} + y = \frac{-1}{3x}$$

$$y = -\frac{1}{3x} - \frac{x-1}{3x} = \frac{-1-(x-1)}{3x} = \frac{\cancel{-1} - x \cancel{+1}}{3x}$$

$$= -\frac{x}{3x} = -\frac{1}{3}$$

$$\frac{-1}{3x} + z = -\frac{3x}{9x} = -\frac{x}{3x}$$

$$z = -\frac{x}{3x} - \frac{-1}{3x} = \frac{-x+1}{3x} = \frac{1-x}{3x}$$

$$-\frac{1}{3} + a = \frac{1-x}{3x}$$

$$a = \frac{1-x}{3x} + \frac{1\cdot x}{3x} = \frac{1\cancel{-x}\cancel{+x}}{3x} = \frac{1}{3x}$$

Aufgabe 5: Vereinfachen Sie die folgenden Doppelbrüche.

a) $\dfrac{\frac{1}{2}}{\frac{1}{2}} = \underline{1} \qquad \left(\frac{1}{2} \cdot \frac{2}{1} = \frac{1 \cdot 2}{2 \cdot 1} = 1 \right)$

b) $\dfrac{\frac{6x}{4}}{\frac{2x}{3}} = \frac{6\cancel{x}}{4} \cdot \frac{3}{2\cancel{x}} = \frac{18}{8} = \underline{\frac{9}{4}}$

c) $\dfrac{\frac{a+b}{a^2-2ab}}{\frac{(a+b)(a-b)}{a}} = \frac{\cancel{(a+b)}}{\underbrace{a^2-2ab}_{\cancel{a}\cdot(a-2b)}} \cdot \frac{\cancel{a}}{\cancel{(a+b)}(a-b)} = \underline{\frac{1}{(a-2b)\cdot(a-b)}}$

d) $\dfrac{\frac{1}{2}+c}{\frac{1}{2}-c} = \frac{1+2c}{\cancel{2}} \cdot \frac{\cancel{2}}{1-2c} = \underline{\frac{1+2c}{1-2c}}$

$\frac{1}{2}+c \rightarrow \frac{1+2c}{2}$

$\frac{1}{2}-c \rightarrow \frac{1-2c}{2}$

e) $\dfrac{1-\frac{1}{2a-1}}{1+\frac{1}{2a-1}} = \dfrac{\frac{(2a-1)-1}{2a-1}}{\frac{(2a-1)+1}{2a-1}} = \frac{2a-2}{\cancel{(2a-1)}} \cdot \frac{\cancel{(2a-1)}}{2a} = \frac{\cancel{2}\cdot(a-1)}{\cancel{2}\cdot a} = \underline{\frac{a-1}{a}}$

f) $\dfrac{\frac{1}{p+2}-\frac{1}{p-2}}{\frac{1}{(p+2)(p-2)}} = \dfrac{\frac{p-2}{(p+2)(p-2)} - \frac{p+2}{(p+2)(p-2)}}{\frac{1}{(p+2)(p-2)}} = \dfrac{\frac{\cancel{p}-2-\cancel{p}-2}{(p+2)(p-2)}}{\frac{1}{(p+2)(p-2)}}$

$= \frac{-4}{\cancel{(p+2)}\cancel{(p-2)}} \cdot \frac{\cancel{(p+2)}\cancel{(p-2)}}{1} = \underline{-4}$